Problem Set 2 – Fundamental of Economics, Data Science for Management, University of Catania.

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(Problem sets should be submitted individually – one for each student – in class on Monday, October 14. Please show not only the solutions but also the relevant steps to obtain the results. Thanks and have fun!)

1. Consider the exponential utiliy function $U = -\exp^{(-\rho c)}$, where c is consumption and $\rho>0$. Show that it is increasing ($u'_c > 0$) and concave ($u_c'' < 0$) for all c as long as $\rho > 0$, that is, as long as the agent is risk-averse. Show that it has constant absolute risk aversion.

2. Consider the power utility function $U = [1/(1-\rho)]\, c^{(1-\rho)}$ with $\rho\neq1$. Show that it is increasing ($u'_c > 0$) and concave ($u_c'' < 0$) for all $c>0$. Show that it has constant relative risk aversion given by ρ.

3. Consider log utility function $U = \log c$. Show that it is increasing ($u'_c > 0$) and concave ($u_c'' < 0$) for all $c>0$. Show that it has constant relative risk aversion given equal to 1.

4. Assume three different individuals with income equal to 30, 20 and 10, respectively. Assume that each of these workers have an utility function $u=c^{2/3} l^{1/3}$, where c is consumption and l is leisure and that the price of consumption is equal to 1. Given the constraint $h = 24 - l$, find the Marshallian demand of consumption, the demand of leisure and the supply of labor. For each of the three workers find the reservation wage.

5. Consider an economic agent that has to invest his wealth *w* in stocks and in bonds. In particular he has to decide the fraction α of his wealth *w* to be invested in stocks and the remaining fraction $1-\alpha$ in bonds. For each euro invested bonds give $(1+r)$, with $r>0$. For each euro invested, stocks give $(1+r_-)$, with $r_- >0$ with probability p and $(1+r_+)$, with $r_+>0$, with probability 1-p.

Write down the maximization problem of this agent assuming a utility function u(w) which is increasing and concave. Precisely, write down the expected utility as a function of α (endogenous variable), p and r, r_+ and r_- (exogenous variables) that should be maximized with respect to α subject to the constraint that $\alpha \in[0,1]$.

Find the first order condition (the derivate of the utility function with respect to α equal to zero).

Verify that the second derivate is always negative (the first derivative of the first order condition).

Use the implicit function theorem to understand how α in the optimum varies with respec to *w* (this is quite difficult).