Problem Set 1 – Fundamental of Economics, Data Science for Management, University of Catania.

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(Problem sets should be submitted individually – one for each student – in class on Monday, October 7. Please show not only the solutions but also the relevant steps to obtain the results. Thanks and have fun!)

Consider an consumer with preferences that can be expressed by the following utility function:

$u(x,y) = \alpha \log x + (1-\alpha) \log y$

the price of good x is p_x and the price of good y is p_y. Instead of having an income M, the consumer has endowments of goods x and y that we indicate with ω_x and ω_y. Since the individual can sell the goods, the endowments are a source of wealth. Hence his income is $p_x\ \omega_x + p_x\ \omega_y$, that should be treated as an exogenous variable.

1. First, write down the budget constraint and plot the budget line assuming that $\omega_x=1$, $\omega_y=1$ and $p_x= p_y=1$. Second, plot the line assuming that p_x increases and is equal to 2. Third, plot the budget constraints for the same combination of prices $p_x= p_y=1$ and $p_x= 2$, $p_y=1$, assuming the standad case with an income M=2 (no endowments). Explain the difference between the variations of the budget constraints with endowments and the one with no endowments and income.
2. Solve the maximization problem and find the Marshallian demand function of x and y as a function of the prices and the endowments.
3. Show how the Marshallian function of the good x varies with p_x, p_y, $\omega_{x,}$ and ω_y.
4. With the Marshallian demand function x* and y*, find the net demands for goods x and y, namely the $z^x = x^* - \omega_x$ and $z^y = y^* - \omega_y$. If a net demand a good is negative, the consumer sells that good. If the net demand is positive, the consumer buys that good. Derive the conditions according to which the net demands are positive and show how the net demands varies with respect to the endowments.
5. Solve the maximization problem and find the Marshallian demand function of x and y as a function of the prices and the endowments under the utility function $u(x,y)= x^{alpha}\ y^{1-alpha}$. Show that you obtain the same results of point 2.
6. Solve the minimization problem min (p_x x + p_xy), subject to the u(x,y)=u, where u(x,y) is the same provided in point 5. Derive the Hicksian (compensated) demand functions of x and y.