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# Sufficientarianism

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Catania, June 7th 2019

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## Goals of this research in progress

Setting: comparisons of allocations of resources, wealth, ... to the members of a society.

We analyse **sufficientarianism** in the context of allocation of opportunities (the latter are conceived of as chances in life).

- ▷ An elementary sufficientarian criterion is characterised which simply counts the number of agents which attain a satisfactory standard of living.
- ▷ A refinement is proposed and fully characterised.
- ▷ Extensions to the intergenerational context, and to the evaluation of infinite streams of opportunities, are considered.

## Sufficientarianism: distinctive features

According to the sufficiency principle, a concern for equality is philosophically misguided and objectionable:

- ▷ Frankfurt “Equality as a Moral Ideal”, *Ethics* 98 (1987): 21–43.
- ▷ Roemer “Eclectic Distributional Ethics”, *Politics, Philosophy and Economics* 3 (2004): 267–281. He explains that sufficientarianism is “the doctrine advising the ethical observer to ‘maximize the number of people who have enough’ in any situation”. Once an agent reaches the threshold, the emphasis is not on making her even better off, but rather on pushing the others up. When all agents are flourishing, for example, it can be argued that distributive concerns are less pressing, so Roemer adds: “distributional ethics are only important when it is possible that some people might not have a good life”.

Appropriate definition of the threshold that identifies “a good life”:  
Income? Welfare?

## A compromise solution: Opportunities as chances in life

Each individual is regarded as a binary experiment with either 'success' or 'failure' as possible outcomes. Then, opportunities in society are expressed by the profile of 'chances of success' across individuals.

This is more convincing than a focus on income but also than a welfarist setting, given the objective nature of the alternatives and the natural scale of measure.

### **Antecedents:**

Mariotti, M. and R. Veneziani (2011) "Allocating chances of success in finite and infinite societies: The Utilitarian criterion", *Journal of Mathematical Economics* 48: 226-236.

Mariotti, M. and R. Veneziani (2018) "Opportunities as chances: maximising the probability that everybody succeeds", *Economic Journal* 128: 1609-1633.

# Table of contents

1. The framework
2. Characterization of the sufficientarian criterion
3. First extension: Multithreshold sufficientarianism
4. Second extension: sufficientarianism for infinite societies
5. Conclusion

# The framework

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## Basic elements I

There are  $\mathcal{T}$  individuals in society.  $\mathcal{T}$  is either a natural number  $T$  or  $\infty$ , interpreted as the cardinalities of a finite set of agents  $\mathcal{N}$  or of an infinite set of agents  $\mathbb{N}$ , respectively

An **opportunity** for individual  $t$  is a number between 0 and 1,  $a_t \in B = [0, 1]$ . It is interpreted as a 'chance of success' either in some given field or in life as a whole, so that opportunities can be manipulated just as probabilities.

We are interested in how opportunities should be allocated among the  $\mathcal{T}$  individuals.

An *opportunity profile* (or simply a **profile**) is a point in the 'box of life'  $B^{\mathcal{T}} = [0, 1]^{\mathcal{T}}$ . In the case of an infinite society,  $B^{\infty}$  denotes the set of countably infinite streams of probabilities of success for agents in  $\mathbb{N}$ . Here we develop the notation for the finite case.

A profile  $a = (a_1, a_2, \dots, a_T) \in B^T$  lists the opportunities, or 'chances of success' of agents in  $\mathcal{N}$  if  $a$  is chosen.

## Basic elements II

The points  $\mathbf{0} = (0, 0, \dots, 0) \in B^T$  and  $\mathbf{1} = (1, 1, \dots, 1) \in B^T$  can be thought of as *Hell* (no opportunities for anybody) and *Heaven* (full opportunities for everybody), respectively.

Let  $B_+^T = \{a \in B^T \mid a \gg \mathbf{0}\}$ .<sup>1</sup>

A **social opportunity relation**  $\succsim$  on  $B^T$  is a binary relation on  $B^T$ .

The relation  $\succsim$  is *reflexive* if, for any  $x \in X$ ,  $x \succsim x$ ; *complete* if, for any  $x, y \in X$ ,  $x \neq y$  implies  $x \succsim y$  or  $y \succsim x$ ; *transitive* if, for any  $x, y, z \in X$ ,  $x \succsim y \succsim z$  implies  $x \succsim z$ .

Given a binary relation  $\succsim$  on a set  $X$  and  $x, y \in X$ , we write  $x \succ y$  (the asymmetric factor) if and only if  $x \succsim y$  and  $y \not\succsim x$ , and we write  $x \sim y$  (the symmetric part) if and only if  $x \succsim y$  and  $y \succsim x$ .

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<sup>1</sup>Vector notation: for all  $a, b \in B^T$  we write  $a \geq b$  to mean  $a_t \geq b_t$ , for all  $t \in \mathcal{N}$ ;  $a > b$  to mean  $a \geq b$  and  $a \neq b$ ; and  $a \gg b$  to mean  $a_t > b_t$ , for all  $t \in \mathcal{N}$ .



# The sufficientarian model

Let  $\alpha \in B$  denote an (ethically determined) threshold identifying a **sufficient** or satisfactory chance of success in life.

For all  $a \in B^T$ :  $P(a, \alpha) = \{i \in \mathcal{N} : a_i \geq \alpha\}$  denotes the set of individuals who have a 'sufficient' chance of success at profile  $a$ .

Let  $n(a, \alpha) = |P(a, \alpha)|$ .

Then, for all  $a, b \in B^T$ :  $a \succcurlyeq_{\alpha}^s b \Leftrightarrow n(a, \alpha) \geq n(b, \alpha)$ .

This criterion incorporates a commitment for equity, in that not even a single additional person below the threshold can be accepted in exchange for any arbitrarily large increase in the opportunities of the others, if this increase does not take at least one other person above the threshold. However, it is silent concerning a number of other potentially relevant tradeoffs.

**Question:** How can we characterise  $\succcurlyeq_{\alpha}^s$ ?

## Basic axioms: efficiency

We aim to specify desirable properties for a social opportunity relation.

Two standard axioms for  $\succcurlyeq$  capturing a notion of efficiency in the allocation of opportunities are the following:

**Strong Pareto:** for all  $a, b \in B^T$ ,  $a > b \Rightarrow a \succ b$ .

**Weak Pareto:** for all  $a, b \in B^T$ ,  $a \gg b \Rightarrow a \succ b$ .

Monotonicity is an extremely weak condition that is satisfied even by orderings that are completely insensitive to efficiency considerations, such as the universal indifference criterion:

**Monotonicity:** for all  $a, b \in B^T$ ,  $a > b \Rightarrow a \succcurlyeq b$ .

## Basic axioms: equal treatment

The next axiom incorporates a notion of fairness by requiring the allocation rule to be insensitive to individual identities.

A *permutation*  $\pi$  is a bijective mapping of  $\mathcal{N}$  onto itself.

**Anonymity:** for all  $a, b \in B^T$ ,  $a = \pi b$  for some permutation  $\pi \Rightarrow a \sim b$ .

## Axiom: NonInterference

A liberal principle of noninterference has been recently proposed by Mariotti and Veneziani (J Econ Theory, 2013).

**NonInterference:** Let  $a, b, a', b' \in B^T$  be such that  $a \succ b$  and, for some  $t \in \mathcal{N}$ ,

$$\begin{aligned}(a_t - a'_t)(b_t - b'_t) &> 0, \\ a_j &= a'_j \text{ for all } j \neq t, \\ b_j &= b'_j \text{ for all } j \neq t.\end{aligned}$$

Then  $b' \not\succeq a'$  whenever  $a'_t > b'_t$ .

▷ If society strictly prefers  $a$  to  $b$ , then it should not reverse its strict preferences in a way that is adverse to any agent when the opportunities of such individual change and all other agents are unaffected.

## Axiom: Independence

The next axiom captures a different form of consistency of social rankings by requiring some independence across individuals:

**Independence:** Let  $a, b, a', b' \in B^T$  be such that for some  $t \in \mathcal{N}$ ,

$$\begin{aligned} a_t = b_t & \quad \text{and} \quad a'_t = b'_t \\ a_j & = a'_j \quad \text{for all } j \neq t, \\ b_j & = b'_j \quad \text{for all } j \neq t. \end{aligned}$$

Then  $a' \succcurlyeq b'$  whenever  $a \succcurlyeq b$ .

▷ Suppose that we compare two distributions where one person  $t$  receives the same amount  $a_t$ , and we replace the opportunities of that person in both distributions with a common new amount  $a'_t$ .

Then provided that nobody else is affected, the comparisons among the distributions are independent of the opportunities of individual  $t$ .

## Axiom: Upper Semicontinuity

Finally, the next axiom captures a technical requirement commonly imposed on social evaluations.

**Upper Semicontinuity:** for all  $a \in B^T$  the set  $\{b \in B^T : a \succ b\}$  is open in  $B^T$ .

## **Characterization of the sufficientarian criterion**

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## Specific axioms: Universal Decency and Avoidance of Penury

We consider two properties recently proposed by Roemer.

Let  $\beta_1, \beta_2, \beta_3 \in B$  represent three pre-specified ethically relevant thresholds of opportunities such that  $\beta_1 \leq \beta_2 \leq \beta_3$ . If the inequalities are strict, they can be interpreted, respectively, as the levels of opportunities associated with a life barely worth living, a mediocre life, and a good life.

**Universal Decency:** for all  $a, b \in B^T : a_t \geq \beta_3$  all  $t$ , and  $b_t < \beta_3$ , some  $t$ ,  $a \succ b$ .

▷ An allocation of opportunities such that all individuals flourish is preferable to one in which only some of them enjoy a good or excellent life.

**Avoidance of Penury:** for all  $a, b \in B^T : a_t \geq \beta_2$ , all  $t$ , and  $b_t < \beta_1$ , some  $t$ ,  $a \succ b$ .

▷ An allocation of opportunities such that all individuals have a decent life is preferable to one in which some of them have a life not worth living.



## Another specific axiom: Weak Universal Decency

The latter two properties assume the existence of pre-specified ethical thresholds. In our analysis, we shall impose a much weaker axiom which only requires the existence of *some* positive threshold.

**Weak Universal Decency:** there exists  $\beta \in (0, 1]$  such that for all  $a, b \in B^T$  :  $a_t \geq \beta$ , all  $t$ , and  $b_t < \beta$ , some  $t$ ,  $a \succ b$ .

Alternative definition of Weak Universal Decency: there exists  $\beta \in (0, 1]$ , such that for all  $a, b \in B^T$ ,  $n(a, \beta) = T > n(b, \beta)$  implies  $a \succ b$ .

In our context, one can conceive of the conjunction of Monotonicity and Weak Universal Decency as a single weakening of the Strong Pareto axiom.

# Properties of the sufficientarian social opportunity relation

The following result gives necessary conditions:

## Proposition

The sufficientarian social opportunity relation  $\succsim_{\alpha}^s$  is an ordering (complete, transitive), and it satisfies Anonymity, NonInterference, Upper Semicontinuity in the Euclidean topology, Monotonicity, Independence, and Weak Universal Decency.

Weak Universal Decency, Independence and NonInterference assure minimal efficiency in the form of the Monotonicity axiom:

## Proposition

Let  $\succsim$  be an ordering on  $B^T$  that verifies Independence, NonInterference, and Weak Universal Decency. Then  $\succsim$  verifies Monotonicity.

## Another property of $\succsim_{\alpha}^s$

The latter proposition allows us to prove that under Independence and NonInterference, Weak Universal Decency is equivalent to the following axiom (which is clearly implied by Weak Universal Decency):

**Axiom E:** there exists  $\beta \in (0, 1]$  such that  $(\beta, \beta, \dots, \beta, \beta) \succ b$  for all  $b \in B^T$  such that  $b_t < \beta$ , some  $t$ , and  $b_j = b_i$ , all  $i, j \neq t$ .

This alternative property incorporates intuitions about both equality and efficiency:

- ▷ There is at least one egalitarian vector –possibly with a very high level of opportunities for everyone– that is preferred to (a certain subset of) vectors with inequalities.
- ▷ Because Axiom E allows for  $\beta = 1$ , it may be interpreted as a strict weakening of Strong Pareto with the same –albeit less pronounced– efficiency flavour.

## Some auxiliary results

The following auxiliary results establish:

Lemma 1: The uniqueness of the ethical threshold when the social opportunity relation verifies certain properties.

Lemma 2: A generalisation of the consequent of NonInterference to any two profiles in which an agent enjoys the same level of opportunities

### Lemma 1.

Let  $\succsim$  be an ordering on  $B^T$  that verifies Independence, NonInterference, Weak Universal Decency (or Axiom E), and Upper Semicontinuity. Then there is a unique  $\beta$  for which Weak Universal Decency holds true.

### Lemma 2.

Let  $\succsim$  be an ordering on  $B^T$  that verifies Independence, NonInterference, and Upper Semicontinuity. Then for any  $a, b \in B^T$ , if  $a \succ b$ , then  $a' \succ b'$  for any  $a', b' \in B^T$  such that  $a'_t = b'_t$  some  $t \in \mathcal{N}$  and  $a'_j = a_j, b'_j = b_j$  for all  $j \neq t$ .

## Further auxiliary results

Now we prove that in the conditions of Lemma 1, the distributions where everyone is above the ethical threshold are all equivalent:

### Lemma 3.

Let  $\succsim$  be an ordering on  $B^T$  that verifies Independence, NonInterference, Weak Universal Decency (for the threshold  $\alpha$ ), and Upper Semicontinuity. Then  $a \sim b$  for all  $a, b \in B^T$  with  $n(a, \alpha) = n(b, \alpha) = T$ .

With these results we can now prove that under the conditions of Lemmas 1 and 3, Anonymity is guaranteed:

### Lemma 4.

Let  $\succsim$  be an ordering on  $B^T$  that verifies Independence, NonInterference, Weak Universal Decency (or Axiom E), and Upper Semicontinuity. Then  $\succsim$  is Anonymous.

# Characterization of the sufficientarian rule

We are ready to establish:

## Characterization of the sufficientarian rule.

The sufficientarian social opportunity relation  $\succsim_{\alpha}^s$  is the only ordering on  $B^T$  that satisfies Independence, NonInterference, Upper Semicontinuity in the Euclidean topology, and Axiom E.

Examples prove that the properties in the Theorem above are independent.

## Violation of equity properties

In order to clarify the foundations of sufficientarianism and the type of intuitions it incorporates, in this section we discuss some standard principles that sufficientarianism does *not* satisfy.

Sufficientarianism is rather insensitive to equity considerations. Consider the following two standard egalitarian axioms:

**Hammond equity:** for all  $a, b \in B^T$  :  $a_i < b_i < b_j < a_j \exists i, j \in \mathcal{N}$ ,  
 $a_k = b_k \forall k \in \mathcal{N} \setminus \{i, j\} \Rightarrow b \succcurlyeq a$ .

**Pigou Dalton:** for all  $a, b \in B^T$ , all  $\delta > 0$ , and all  $i, j \in \mathcal{N}$ ,  $a_k = b_k$   
 $\forall k \in \mathcal{N} \setminus \{i, j\}$ ,  $a_i = b_i - \delta \geq b_j + \delta = a_j \Rightarrow a \succcurlyeq b$ .

To see these two facts, let  $\alpha = 1/2$  and consider two profiles  $a, b \in B^T$  such that  $a = (\frac{5}{8}, 0, 1, 1, 1, \dots, 1)$  and  $b = (\frac{3}{8}, \frac{1}{4}, 1, 1, 1, \dots, 1)$ . By definition  $a \succcurlyeq_{\alpha}^s b$ , which violates both axioms.

## Violation of continuity

Already Roemer had observed that sufficientarianism contradicts lower semicontinuity.

Let  $\alpha \in (0, 1)$ , and consider  $a, b \in B^T$  such that  $a_k = \alpha$  for all  $k \in \mathcal{N}$ , and  $b_1 = 1$ ,  $b_k = 0 \forall k \in \mathcal{N} \setminus \{1\}$ , and  $\mathcal{T} > 1$ . Clearly,  $a \succ_{\alpha}^s b$ , but there exists no neighbourhood  $\mathcal{B}(a, \delta)$  of  $a$  such that  $a' \succ_{\alpha}^s b$  for all  $a' \in \mathcal{B}(a, \delta)$ , both in the supremum and the Euclidean topology.



## **First extension: Multithreshold sufficientarianism**

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## Distinctive features and definition

The sufficientarian criterion gives rise to very large indifference classes. This refinement addresses this issue in an ethically relevant way.

Formally, let  $\alpha, \alpha' \in B$  denote two (ethically determined) distinct thresholds with  $\alpha > \alpha'$ , identifying, respectively, a *satisfactory* chance of success in life, and a *minimum* chance of leading a life worth living.

A natural, multithreshold extension of the simple sufficientarian ordering, the **multithreshold sufficientarian** relation,  $\succ_{\alpha, \alpha'}^s$ , is as follows.

For all  $a, b \in B^T$ :

$a \succ_{\alpha, \alpha'}^s b \Leftrightarrow$  either  $n(a, \alpha') > n(b, \alpha')$  or  
 $n(a, \alpha') = n(b, \alpha')$  and  $n(a, \alpha) > n(b, \alpha)$

$a \sim_{\alpha, \alpha'}^s b \Leftrightarrow n(a, \alpha') = n(b, \alpha')$  and  $n(a, \alpha) = n(b, \alpha)$

**Ethical thresholds:** there exist  $\beta, \beta' \in (0, 1)$ , with  $\beta > \beta'$  such that for all  $a, b \in B^T$ :

(i)  $a_t \geq \beta'$ , all  $t$ , and  $b_t < \beta'$ , some  $t$ , imply  $a \succ b$ ; and

(ii)  $a_t \geq \beta$ , all  $t$ , and  $b_t < \beta$ , some  $t$ , imply  $a \succ b$ .

It is not difficult to see that Multithreshold sufficientarianism clashes with a complete liberal view of NonInterference. It is however compatible with some limited liberal notions of protection from interference analogous to the Harm Principle and the Benefit Principle:

## Adapted axiomatics II

The core of the next axiom is the requirement that an individual who has suffered damage without harming others should not be interfered with (it is libertarian rather than egalitarian) under some given circumstances:

**$(\beta, \beta')$ -Restricted Harm Principle:** Let  $a, b, a', b' \in B^T$  be such that  $a \succ b$  and, for some  $t \in \mathcal{N}$ ,

$$a_t > a'_t,$$

$$b_t > b'_t,$$

$$a_j = a'_j \text{ for all } j \neq t,$$

$$b_j = b'_j \text{ for all } j \neq t.$$

Then  $b' \not\prec a'$  whenever  $a'_t > b'_t$  and either  $n(a', \beta) = n(a, \beta)$ , or  $n(a', \beta') = n(a, \beta')$ , or both.

## Adapted axiomatics III

Similarly, the core of the next libertarian axiom is the requirement that an individual who benefits without affecting others should not be interfered with:

**$(\beta, \beta')$ -Restricted Benefit Principle:** Let  $a, b, a', b' \in B^T$  be such that  $a \succ b$  and, for some  $t \in \mathcal{N}$ ,

$$a_t < a'_t,$$

$$b_t < b'_t,$$

$$a_j = a'_j \text{ for all } j \neq t,$$

$$b_j = b'_j \text{ for all } j \neq t.$$

Then  $b' \not\prec a'$  whenever  $a'_t > b'_t$ ,  $n(b, \beta) = n(b', \beta)$ , and  $n(b, \beta') = n(b', \beta')$ .

# Characterization of multithreshold sufficientarian rule

The relation  $\succ_{\alpha, \alpha'}^s$  is an ordering, and it satisfies Anonymity,  $(\alpha, \alpha')$ -Restricted Harm Principle,  $(\alpha, \alpha')$ -Restricted Benefit Principle, Upper Semicontinuity in the Euclidean topology, Monotonicity, Independence, and Ethical Thresholds.

Multi threshold sufficientarianism also incorporates a basic principle of fairness. It allows for some concern for efficiency.

It implements some independence across individuals and a limited form of continuity in ethical judgements. It also incorporates some (restricted) liberal intuitions concerning autonomy and protection from interference.

Conversely, we are ready to establish:

## Characterization of the multithreshold sufficientarian rule.

The sufficientarian social opportunity relation  $\succ_{\alpha, \alpha'}^s$  is the only ordering on  $B^T$  that satisfies Anonymity, Monotonicity, Independence,  $(\alpha, \alpha')$ -Restricted Harm Principle,  $(\alpha, \alpha')$ -Restricted Benefit Principle, Upper Semicontinuity in the Euclidean topology, and Ethical Thresholds.

**Second extension:  
sufficientarianism for infinite  
societies**

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## Distinctive features

The previous notation is extended in a straightforward way to the infinite context, with the following specific additions.

A profile is now denoted  ${}_1a = (a_1, a_2, \dots) \in B^\infty$ , where  $a_t$  is the probability of success of generation  $t \in \mathbb{N}$ . For  $T \in \mathbb{N}$ ,  ${}_1a_T = (a_1, \dots, a_T)$  denotes the  **$T$ -head** of  ${}_1a$  and  ${}_{T+1}a = (a_{T+1}, a_{T+2}, \dots)$  denotes its  **$T$ -tail**, so that  ${}_1a = ({}_1a_T, {}_{T+1}a)$ .

For any  $x \in B$ ,  $\mathbf{x} = (x, x, \dots) \in B^\infty$  denotes the stream of constant probabilities equal to  $x$ . Let  $B_+^\infty = \{{}_1a \in B^\infty \mid {}_1a \gg \mathbf{0}\}$ . For all  ${}_1a \in B^\infty$  and  $T \in \mathbb{N}$ , let  $P^{1a_T} = \{t \in \{1, \dots, T\} : a_t > 0\}$ .

A *permutation*  $\pi$  is now a bijective mapping of  $\mathbb{N}$  onto itself. A permutation  $\pi$  of  $\mathbb{N}$  is finite if there is  $T \in \mathbb{N}$  such that  $\pi(t) = t$ , for all  $t > T$ , and  $\Pi$  is the set of all finite permutations of  $\mathbb{N}$ . For any  ${}_1a \in B^\infty$  and any  $\pi \in \Pi$ , let  $\pi({}_1a) = (a_{\pi(t)})_{t \in \mathbb{N}}$  be a permutation of  ${}_1a$ . For any  ${}_1a \in B^\infty$ , let  ${}_1\bar{a}_T$  denote the permutation of the  $T$ -head of  ${}_1a$ , which ranks the elements of  ${}_1a_T$  in ascending order.



## Definition of sufficientarian overtaking criterion

**The sufficientarian overtaking criterion:** For all  ${}_1a, {}_1b \in B^\infty$ ,

$${}_1a \succ_{\alpha}^{s*} {}_1b \Leftrightarrow \exists \tilde{T} \in \mathbb{N} \text{ such that } \forall T \geq \tilde{T} : n({}_1a_T, \alpha) > n({}_1b_T, \alpha),$$

$${}_1a \sim_{\alpha}^{s*} {}_1b \Leftrightarrow \exists \tilde{T} \in \mathbb{N} \text{ such that } \forall T \geq \tilde{T} : n({}_1a_T, \alpha) = n({}_1b_T, \alpha).$$

Next, we reformulate our main axioms to hold in the infinite context.

## Adapted axiomatics I

**NonInterference:** Let  ${}_1a, {}_1b \in B^\infty$  be such that  ${}_1a = ({}_1a_{T, T+1} b)$  for some  $T \in \mathbb{N}$  and  ${}_1a \succ {}_1b$ ; and  ${}_1a', {}_1b' \in B^\infty$ , for some  $t \in \mathbb{N}$ ,

$$\begin{aligned}(a_t - a'_t)(b_t - b'_t) &> 0, \\ a_j &= a'_j \text{ for all } j \neq t, \\ b_j &= b'_j \text{ for all } j \neq t.\end{aligned}$$

Then  $b' \not\prec a'$  whenever  $a'_t > b'_t$ .

**Independence:** Let  ${}_1a, {}_1b, {}_1a', {}_1b' \in B^\infty$  be such that  ${}_1a = ({}_1a_{T, T+1} b)$  for some  $T \in \mathbb{N}$  and for some  $t \in \mathbb{N}$ ,

$$\begin{aligned}a_t = b_t \quad \text{and} \quad a'_t = b'_t \\ a_j = a'_j \text{ for all } j \neq t, \\ b_j = b'_j \text{ for all } j \neq t.\end{aligned}$$

Then  $a' \succcurlyeq b'$  whenever  $a \succcurlyeq b$ .

## Adapted axiomatics II

**Upper Semicontinuity:** for all  ${}_1a \in B^\infty$  the set  $\{{}_1b \in B^\infty : {}_1a \succ {}_1b\}$  is open in  $B^\infty$ .

**Weak Universal Decency:** there exists  $\beta \in (0, 1]$  such that for all  ${}_1a, {}_1b \in B^\infty$  such that  ${}_1a = ({}_1a_T, {}_{T+1}b)$  for some  $T \in \mathbb{N}$ :  $a_t \geq \beta$ , all  $t \leq T$ , and  $b_t < \beta$ , some  $t \leq T$ ,  ${}_1a \succ {}_1b$ .

**Axiom E:** there exists  $\beta \in (0, 1]$  such that for all  ${}_1a, {}_1b \in B^\infty$  such that, for some  $T \in \mathbb{N}$ ,  $a_t = \beta$  and  ${}_1a = ({}_1a_T, {}_{T+1}b)$ :  $b_t < \beta$ , some  $t$ , and  $b_j = b_i$ , all  $i, j \neq t$  implies  ${}_1a \succ {}_1b$ .

The next axiom represents a mainly technical requirement to deal with infinite dimensional profiles (Asheim and Tungodden, Econ Theory 2004).

**Preference Continuity:** for all  ${}_1a, {}_1b \in B^\infty$ ,  ${}_1a \succ {}_1b$  whenever  $\exists \tilde{T} \geq 1$  such that  $({}_1a_T, {}_{T+1}b) \succ {}_1b \forall T \geq \tilde{T}$ .

## Adapted axiomatics III

Preference continuity provides a condition that establishes “a link to the standard finite setting of distributive justice, by transforming the comparison of any two infinite utility paths to an infinite number of comparisons of utility paths each containing a finite number of generations” (Asheim and Tungodden, Econ Theory 2004).

In the same vein, the next axiom states that the relation  $\succsim$  on  $B^\infty$  should at least be able to compare (infinite-dimensional) probability profiles with the same tail. This seems an obviously desirable property which imposes a minimum requirement of completeness.

**Minimal Completeness:** for all  ${}_1a, {}_1b \in B^\infty$ ,  $\exists T \geq 1$  ( ${}_1a_{T, T+1} b$ )  $\neq$   ${}_1b \Rightarrow ({}_1a_{T, T+1} b) \succsim {}_1b$  or  ${}_1b \succsim ({}_1a_{T, T+1} b)$ .

Lombardi and Veneziani, Econ J 2016, use Minimal Completeness to characterise the infinite leximin and maximin social welfare relations.

The next Theorem proves that the combination of NonInterference, Weak Universal Decency, Independence, Upper Semicontinuity, Preference Continuity and Minimal Completeness characterises the set of extensions of the sufficientarian overtaking criterion:

## Characterization of the sufficientarian overtaking criterion.

A quasi-ordering  $\succsim$  on  $B^\infty$  is an extension of  $\succsim_\alpha^{s^*}$  if and only if  $\succsim$  on  $B^\infty$  satisfies NonInterference, Weak Universal Decency, Independence, Upper Semicontinuity, Preference Continuity and Minimal Completeness.

## Conclusion

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# Summary

Sufficientarianism has found support from political philosophers: society is best when the highest number of people reach a satisfactory welfare level (or a good enough chance of success in life: we benefit from the objective nature of the alternatives and the natural scale of measure).

However sufficientarianism is relatively unexplored in normative economics and social choice theory.

We have provided axiomatic basis for sufficientarianism when the society is finite.

We have also hinted at two extensions:

- ▷ A refinement in the same spirit where the indifference classes are less thick (i.e., it allows for finer discriminations).

Alternative for future consideration: when an equal number of people are above the threshold at two profiles, another criterion (e.g., utilitarian) decides which of them is better.

- ▷ An extension to the case of infinite societies.



**Thank you!**